Math 511

1.2/1.3 Gauss and Gauss Jordan Elimination — solve sys. of eqn's

row ech. and reduced row ech.

Matrix Arith

Q

$$\xrightarrow{6} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -7 & -6 & -10 \\ 0 & -1 & -1 & -2 \end{array}\right] \quad r_2\left(-\tfrac{1}{7}\right) + r_3 = \text{New } r_3$$

vs.

$$\xrightarrow{2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 7 & 6 & 10 \end{array}\right]$$

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strict triangular

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow{\text{back sb.}} \boxed{2z = 6} \quad z = 3$$

$y + z = 2$

$y = -1$

$x + 2y + 3z = 4 \longrightarrow x = -3$

$\boxed{(-3, -1, 3)}$

Q what if you don't get strict triangular?

→ goal: row echelon form

$$[A|b] = \left[\begin{array}{cccc|c} x & x & \cdots & x & x \\ 0 & x & \cdots & x & \vdots \\ 0 & 0 & x & \cdots & \vdots \\ 0 & 0 & 0 & x & x \end{array}\right]$$

strict triangular

(also row ech)

1	2	3	4
0	1	2	3
0	0	1	2
0	0	0	1

↓

1	2	3	4
0	0	1	2
0	0	0	1
0	0	0	0

If you make a matrix in row ech. → Gaussian Elimination

Solve: System of eqns

- Soln → Consistent
- No Soln → Inconsistent

ex) 2D graph

- 1 soln
- or ∞ soln
- 0 solns

① Inconsistent

ex

row-ech →

1	2	3	4	5
0	0	2	3	1
0	0	0	0	4

∴ no soln eqn

so no soln sys soln

② Consistent

a) one soln

row ech → [strict triangular]

b) ∞ soln

row ech →

	x_1	x_2	x_3	x_4	
ex →	1	2	2	1	3
	0	0	1	1	1
	0	0	0	3	6

leads $\{x_1, x_3, x_4\}$

free $\{x_2\}$

1st non-zero called lead variable

not a lead? → call it a free variable

$x_2 = a$ $x_4 = 2$ $x_3 = -1$ $x_1 = 3 - 2a$

$m \times n$ systems

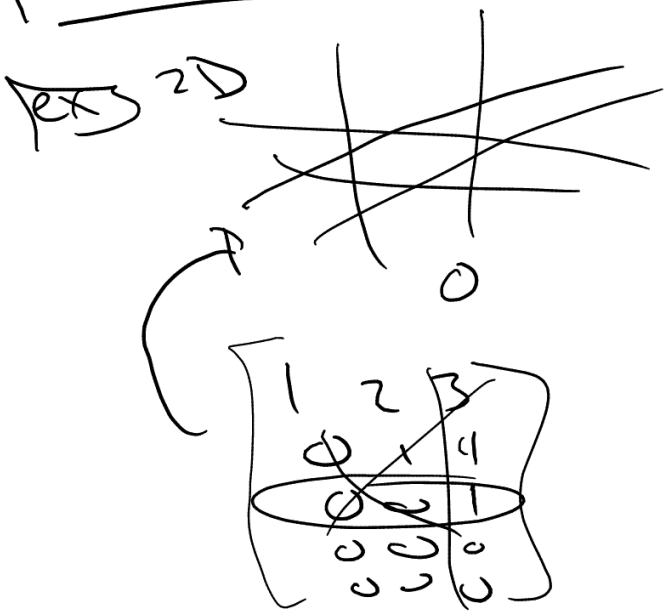
$m > n$ overdetermined

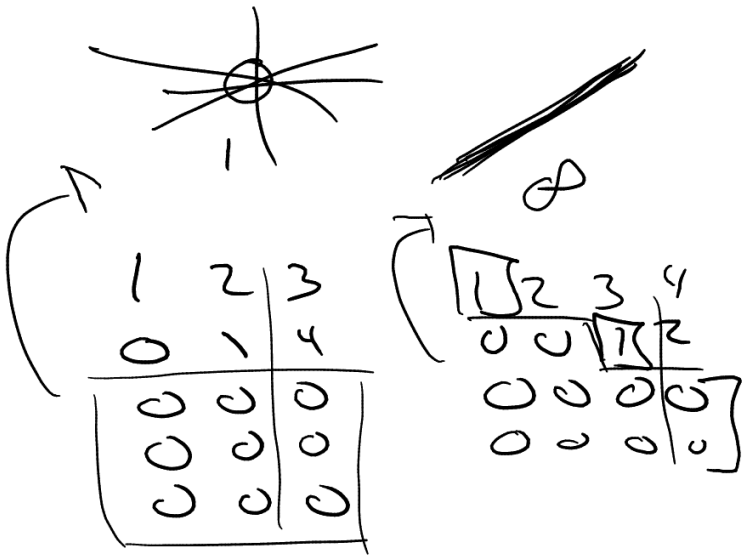
$m < n$ underdetermined

$m = n$ determined

overdetermined

ex) 2D





1	2	3
0	1	4
0	0	0
0	0	0
0	0	0

1	2	3	4
0	0	1	2
0	0	0	0
0	0	0	0

under det

no soln or ∞ soln

det (all 3 possible)

Skip back solve?

↳ Gauss – Jordan Elimination

↑ Row-ech

↑ (reduced) Row-ech

ex)

1	2	3	4
0	0	7	2
0	0	0	0

ex)

1	2	0	0	1
0	0	1	0	0
0	0	0	1	1

Note: system of eqn's has all zeros on the right

(2x) $2x + y = 0$

$x - y = 0$

call it a **Homogeneous system of eqn's**

Matrices

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Matrix Arithmetic (1.3)

Symbols

$$A_{m \times n} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} = [a_{ij}]$$

Ops

① Same?

$A = B$ when $a_{ij} = b_{ij}$ and both $m \times n$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

(2) $A + B = [a_{ij} + b_{ij}]$

(ex) $(2x + 3y) + (4x + 7y)$

$[2 \quad 3] + [4 \quad 7]$

(3) $\alpha A = [\alpha a_{ij}]$

(ex) $3(2x + y) = 6x + 3y$

(4) $A - B = A + (-1)B = [a_{ij} - b_{ij}]$

Note Identity vs Inverse on Operation

$A + B$

$A + \bigcirc = A$

↑ all zero matrix (Identity)

$A + (-1)A = \bigcirc$

↑ (Inverse)

Matrix · Matrix

↑ times?

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 15 \end{bmatrix}$$

2×3 $\quad$ 3×1 $\quad$ 2×1

$$2x + 3y = 4$$

$$\underset{1 \times 2}{\begin{bmatrix} 2 & 3 \end{bmatrix}} \underset{2 \times 1}{\begin{bmatrix} x \\ y \end{bmatrix}} = \begin{bmatrix} 4 \end{bmatrix}$$

Scalar prod.