Math 511

Q's | 1.2 #9

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right]$$

trivial solution

a) homogeneous → must have at least $(0,0,0)$ as a solution

Not possible to be inconsistent.

b)
$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & 1+\beta & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & -2+\beta & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} \boxed{1} & 2 & 1 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{0} & 0 \end{array}\right] \quad \text{if } \boxed{\beta = 2}$$

Matrix: $\underset{m\times n}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & & \cdots & a_{mn} \end{bmatrix}$

$\underset{1\times n}{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$ row vector $\vec{v}$

$\underset{m\times 1}{A} = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ col. vector (or just vector) v

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Matrix · Matrix ?

① Scalar product (row vector)(col vector) = scalar

$$\underset{1\times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}} = 1\cdot 1 + 2\cdot(-1) + 3\cdot 2 = 5$$

$$\boxed{\begin{array}{c} ax + by = c \\ \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = c \end{array}}$$

① Vector · Vector $\qquad \vec{a}_1\, \mathbb{x} = c$

② Matrix · vector

$$\boxed{a_{11}x_1 + a_{12}x_2 + a_{13}x_3} = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$\underset{3\times 3}{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}}$$

$$\begin{bmatrix} \vec{a}_1\, \mathbb{x} \\ \vec{a}_2\, \mathbb{x} \\ \vec{a}_3\, \mathbb{x} \end{bmatrix}$$

Def: $\quad A = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_m \end{bmatrix}$

$$\boxed{A\mathbb{x} = \begin{bmatrix} \vec{a}_1\, \mathbb{x} \\ \vec{a}_2\, \mathbb{x} \\ \vdots \\ \vec{a}_m\, \mathbb{x} \end{bmatrix}}$$

or $A = [\mathbf{a}_1 \; \mathbf{a}_2 \cdots \mathbf{a}_n]$

$$\boxed{A\mathbf{x} = x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + \cdots + x_n \mathbf{a}_n}$$

linear combination of A's col. vectors

useful? Solve: $A\mathbf{x} = \mathbf{b}$

$$A\mathbf{x} = \mathbf{0}$$

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③ Matrix · Matrix?

$$\underset{m\times n}{A} \cdot \underset{n\times k}{B} = A[\mathbf{b}_1 \; \mathbf{b}_2 \cdots \mathbf{b}_k]$$

$$= \underset{m\times k}{[A\mathbf{b}_1 \;\; \underbrace{A\mathbf{b}_2}_{\substack{m\times n \; n\times 1 \\ m\times 1}} \;\; A\mathbf{b}_3 \;\; \cdots \;\; A\mathbf{b}_k]}$$

$\vec{a}_1 \mathbf{b}_1$
$\vec{a}_2 \mathbf{b}_1$
$\vdots$
$\vec{a}_m \mathbf{b}_1$

$\vec{a}_i \mathbf{b}_j$

$$\underset{m\times n}{A}\;\underset{n\times k}{B} = \underset{m\times k}{C} = [c_{ij}]$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 5 \\ 2 & 6 & 10 \end{bmatrix}$$

$2\times 2 \quad 2\times 3$

$\boxed{\text{Ops}}$ $A+B$, αA, $A-B$, $\vec{a_i}\, b$

$$A\mathbf{x} = \begin{bmatrix} \vec{a_1}\mathbf{x} \\ \vec{a_2}\mathbf{x} \\ \vdots \\ \vec{a_n}\mathbf{x} \end{bmatrix} = x_1 a_1 + x_2 a_2 + \dots + x_n a_n$$

$$A \cdot B$$

$\boxed{\text{Transpose}}$ $A = [a_{ij}]$

$$A^T = [a_{ji}]$$

if $A^T = A$ we call A symmetric

1.4 Linear Algebra

Laws? $\boxed{\text{th}^{m}\ 1.4.1}$

1. $A + B = B + A$
2. $(A+B)+C = A+(B+C)$

is $\underset{1\times 3\ \ 3\times 4}{AB}$ vs $\cancel{\underset{3\times 4\ \ 1\times 3}{BA}}$ the same?

Matrix Mult. is $\boxed{\text{not}}$ commutative

3. $(AB)C = A(BC)$
4. $A(B+C) = AB + AC$
5. $(A+B)C = (AC+BC)$

(6) $(\alpha\beta)A = \alpha(\beta A)$

(7) $\alpha(AB) = (\alpha A)B = A(\alpha B)$

(8) $(\alpha+\beta)A = \alpha A + \beta A$

(9) $\alpha(A+B) = \alpha A + \alpha B$

Transpose: (1) $(A^T)^T = A$

(2) $(\alpha A)^T = \alpha A^T$

(3) $(A+B)^T = A^T + B^T$

(4) $(AB)^T = B^T A^T$

Note: B/c $(AB)C = A(BC)$

$(n\times n)$

consider $AAAA = A^4$

$A^1 = A$

$A^2 = A\cdot A$

$A^3 = A\cdot A\cdot A = \underline{A^2\cdot A} = A\cdot A^2$

$A^n = A^{n-1}\cdot A$

Identity / Inverse

$\boxed{A+B}$

$A + \emptyset = A$ ← all zero matrix (Add. Identity)

$A + \boxed{(-1)A} = \emptyset$ ← Add. inv. of A.

$\boxed{A \cdot B}$

Identity $\quad I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$

$$I = \begin{bmatrix} 1 & 0 & 0 & & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & & & & 1 \end{bmatrix}$$

$\uparrow \quad \uparrow$

$e_1 \quad e_2$

$$\underset{m \times n}{A} \; \underset{n \times n}{I} = A$$

$$\underset{m \times m}{I} \; \underset{m \times n}{A} = A$$

Inverse?

(1) Needs to be commutative.

$$A \cdot (A\text{'s inv}) = I$$

$$(A\text{'s inv}) \cdot A = I$$

$\rightarrow$ So A must be $\underline{\underline{n \times n}}$

(2) it has to actually mult to I

$\underline{\underline{\text{for}}}$ $\boxed{\text{now}}$ we can only $\underline{\text{check}}$ if two matrices are inverses of each other.

$\boxed{Q}$ are A, B inv?

check : $AB = BA = I$

Yes? $\rightarrow$ inv.

No? $\rightarrow$ Not inv.

Def $A_{n \times n}$ is called non-singular if it has an inverse.

$A_{n \times n}$ is called $\boxed{\text{singular}}$ if it doesn't have an inverse.

Idea:

$$\boxed{A}X = B$$

$$\boxed{A^{-1}A}X = A^{-1}B$$

$$IX = A^{-1}B$$

$$X = A^{-1}B$$

$$3X = 2$$

$$\frac{1}{\cancel{3}}\cancel{3}X = \frac{1}{3}\cdot 2$$

$$1 \cdot X = \frac{2}{3}$$

$$X = \frac{2}{3}$$