Math 511

$\det(A) =$ tech #1 cofactor expansion along any row / col

tech #2 reduce A to U (upper triangular)

using elem. row ops (elem. matrices)

$\to$ $\boxed{E_k \cdots E_2 E_1 A = U}$

$\det(E_k \cdots E_2 E_1 A) = \det(U)$

$$\to \boxed{\det(A) = \frac{\det(U)}{\det(E_k) \cdots \det(E_2)\det(E_1)}}$$

$\boxed{\text{when}}$ $E_k \cdots E_2 E_1 A = U$

$\to$ Have $\det(E_k) \cdots \det(E_2)\det(E_1)\det(A) = \det(U)$

p. 61 $\boxed{\text{thm } 1.5.2}$ A^{-1} exists

(A is non-singular)

$\equiv$ ($A\mathbf{x} = \mathbf{0}$ has only trivial soln $\mathbf{x} = \mathbf{0}$)

$\equiv$ (A is row equiv to I)

Idea: A

$A \to E_1 A \to E_2 E_1 A \to \cdots \boxed{(E_k \cdots E_2 E_1 A) = U}$

$\to \cdots (E_n \cdots E_2 E_1 A) = I$

Note: U can go to I if U has non-zero values on the diagonal

$\det(U) \neq 0$ then U goes to I

$\det(U) = 0$ then U can not go to I

b/c $\underbrace{\boxed{\det(E_k) \cdots \det(E_2)\det(E_1)\det(A)}}_{\text{all not zero}} = \det(U)$

$\det(U) = 0$

$\rightarrow \det(A) = 0$

Thm A is singular iff $\det(A) = 0$

Same as

A is non-singular iff $\det(A) \neq 0$

2.3

given A we can make a matrix of all A_{ij} cofactors (of a_{ij})

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{n1} & & \cdots & a_{nn} \end{bmatrix} \qquad \text{cofactor Matrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ \vdots & & & \\ A_{n1} & & \cdots & A_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} +|M_{11}| & -|M_{12}| & \cdots \\ -|M_{21}| & \cdots & \\ & & \end{bmatrix}$$

Def adjoint $\operatorname{adj}(A) = (\text{cofactor matrix})^T$

$$\operatorname{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \vdots & & & \\ A_{n1} & & \cdots & A_{nn} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & & A_{nn} \end{bmatrix}$$

Thm

$$A \operatorname{adj}(A) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \vdots \\ a_{n1} & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & & \vdots \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & & A_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} \det(A) & & & 0 \\ & \det(A) & & \\ & & \ddots & \\ 0 & & & \det(A) \end{bmatrix}$$

$$= \det(A) \cdot I$$

So $\underline{(A)}\underline{\left(\frac{1}{\det(A)} \cdot \operatorname{adj} A\right)} = \underline{I}$

So $\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$

Thm Cramer's Rule

A is non-singular, $\mathbb{b} \in \mathbb{R}^n$

define A_i to be the new matrix formed by replacing the i^{th} col. of A with $\mathbb{b}$.

then the uniq soln to $A\mathbb{x} = \mathbb{b}$

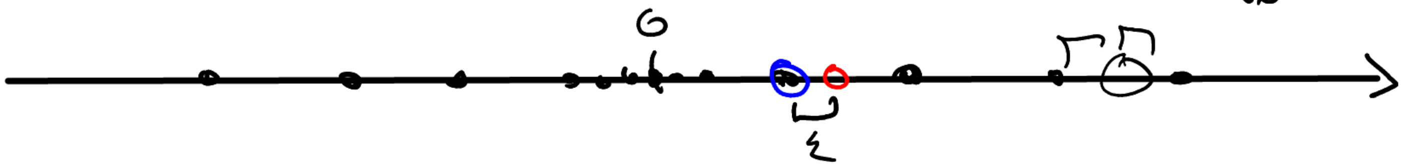
$$\mathbb{x} = \frac{1}{\det(A)} \begin{bmatrix} \det(A_1) \\ \det(A_2) \\ \vdots \\ \det(A_n) \end{bmatrix}$$

Note: can $\det(A)$ even be found computationally?

→ Gaussian Elimination.

want $\det(A) \neq 0$ vs $\det(A) = 0$

in floating point numbers   ~~8~~ bit $2^{\cancel{3}} = 8$ total / 16 numbers



$-4\left(\frac{1}{3}\right) r_1 + r_2 = \text{new } r_2$ → 3 2 1   4

$\boxed{\text{Exam 1}}$ 12 probs @ 10 pts.

→ $\boxed{110 \text{ pts} = 100\%}$

1.1 Systems of linear eqns (2 probs)

① Solve w/o Matrices

② Solve with Aug. Matrices

1.2 Row / Reduced Row Echelon (1 prob)

① gauss-jordan — be able to do 1 soln, ∞ soln, $\emptyset$ soln probs.

1.3 Matrix Arithmetic (1 prob with several parts)

① αA, $A+B$, AB, A^n, A^T

$\boxed{1.4}$ Matrix Algebra (1 prob with parts)

① ops with moving around. $A^{-1} = \ldots$

$$AX + B = C^3 \qquad A = \ldots$$

$$B = \ldots$$

$$X = ? \qquad C = \ldots$$

$\boxed{1.5}$ Elementary Matrices (4 probs)

① $A \to LU$ factorization

② $A \to A^{-1}$

③ Know thm 1.5.2 (State it)

(+) be able to use it.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & -1 & \alpha \end{bmatrix}$$

when is A singular

when is A non-singular

(what $\alpha = ?$)

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 4+\alpha \end{bmatrix}$$

④ Solve eqn's using matrices.

$\boxed{1.6}$ Partitioned Matrices (1 prob)

① Ops $\boxed{ex}$

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \end{bmatrix} \begin{bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{bmatrix} = \begin{bmatrix} C_{11} \\ C_{21} \end{bmatrix}$$

3x5 3x2 3x5 2x3 3x1 2x1

$\boxed{2.1/2.2}$ Determinants 2 probs

① det(A) by cofactors
② det(A) by elimination

and
ans. questions
using thm 2.2.2

det(A) = 0 ?
det(A) ≠ 0 ?

$\boxed{2.3}$ ① probs