Math 321

$$\begin{aligned} x + y + u &= 4 \\ 2y + u &= 4 \end{aligned}$$

subtract → $x - y = 0 \rightarrow \underline{x = y}$

$\underline{u} = 4 - \underline{2y}$

$$\begin{aligned} x - 2y + z + u &= 2 \\ 2x + y + z - u &= 2 \end{aligned}$$

$$\begin{aligned} y - 2y + z + (4 - 2y) &= 2 \\ 2y + y + z - (4 - 2y) &= 2 \end{aligned}$$

$$\begin{aligned} z - 3y &= -2 \\ z + 5y &= 6 \end{aligned}$$

$-8y = -8 \rightarrow y = 1,\ x = 1,\ z = 1,\ u = 2$

$$\boxed{(1, 1, 1, 2)}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 4 \\ 0 & 2 & 0 & 1 & 4 \\ 1 & -2 & 1 & 1 & 2 \\ 2 & 1 & 1 & -1 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 4 \\ 0 & 2 & 0 & 1 & 4 \\ 0 & -3 & 1 & 0 & -2 \\ 0 & -1 & 1 & -3 & -6 \end{array}\right] \rightarrow \underline{\text{etc}}$$

Thurs → Tues

$\frac{80}{120} \rightarrow 120$

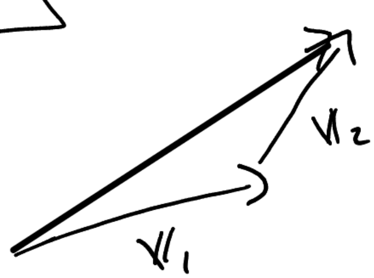
Fix exam for 50% of missed points

Ch3

Sets

S = { collection of elements }

"Space"

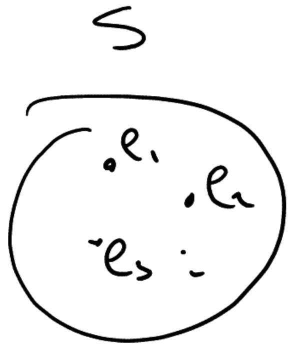


$v_1 + v_2$

αv_1

Set

representations.



① list

② Set Builder (Logical Function)

③ Induction

(Basis) a) Basic Start Elements

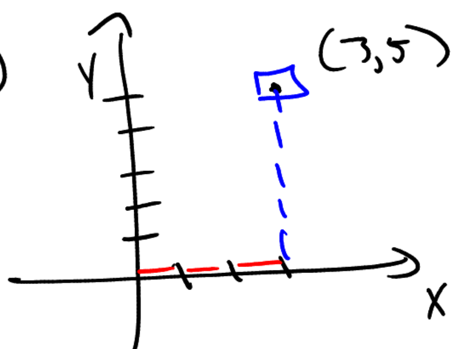
(Inductive step) b) Rules to make new elements in the set

ex $3 \in S$ (Basis)

(Inductive Step) give integer addition when e_1, e_2 are in the Set

$e_1 + e_2$ is in the set

2D

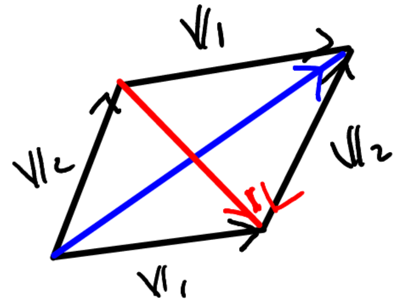


"Vector" Spaces

$\boxed{2D}$ $\quad \mathbb{V} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

define $\mathbb{V}_1 + \mathbb{V}_2$ to be our normal matrix/vector addition

define $\alpha \mathbb{V}_1$ to be our normal scalar multiplication



$\mathbb{V}_1 + \mathbb{V}_2$

$\mathbb{V}_1 - \mathbb{V}_2 \quad \left(= \mathbb{V}_1 + (-\mathbb{V}_2) \right)$

$$\alpha \mathbb{V}_1 = \begin{bmatrix} \alpha v_1 \\ \alpha v_2 \end{bmatrix}$$

$$|\alpha \mathbb{V}_1| = \sqrt{(\alpha v_1)^2 + (\alpha v_2)^2} = |\alpha| \sqrt{v_1^2 + v_2^2}$$
$$= |\alpha| \, |\mathbb{V}_1|$$

We can go to n^{th} Dimensional Space.. with normal ch 1-2 definitions

$$\mathbb{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad \left. \begin{matrix} \mathbb{V}_1 + \mathbb{V}_2 \\ \alpha \mathbb{V}_1 \end{matrix} \right\}$$

<u>Vector Space</u>

Set of all vectors $\mathbb{V}$ — V

given $\mathbb{V}_1 + \mathbb{V}_2$, $\alpha \mathbb{V}_1$

① Closure

(1) $\mathbb{V}_1 + \mathbb{V}_2$ is in V

(2) $\alpha \mathbb{V}_1$ is in V

(2) Properties of $\mathbb{V}_1 + \mathbb{V}_2$

A1) $\mathbb{V}_1 + \mathbb{V}_2 = \mathbb{V}_2 + \mathbb{V}_1$]

A2) $(\mathbb{X} + \mathbb{Y}) + \mathbb{Z} = \mathbb{X} + (\mathbb{Y} + \mathbb{Z})$]

A3) $\mathbb{O} \in V$ where $\mathbb{V}_1 + \mathbb{O} = \mathbb{V}_1$]

A4) $-\mathbb{X}$ exists where $\mathbb{X} + (-\mathbb{X}) = \mathbb{O}$]

(5) Properties of $\alpha \mathbb{V}_1$

A5) $\alpha(\mathbb{V}_1 + \mathbb{V}_2) = \alpha \mathbb{V}_1 + \alpha \mathbb{V}_2$]

A6) $(\alpha + \beta)\mathbb{V}_1 = \alpha \mathbb{V}_1 + \beta \mathbb{V}_1$]

A7) $(\alpha \beta)\mathbb{V}_1 = \alpha(\beta \mathbb{V}_1)$]

A8) $1 \cdot \mathbb{V}_1 = \mathbb{V}_1$]

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extra properties that follow the above <u>10</u>

$\boxed{\text{Thm}}$

(1) $0\,\mathbb{X} = \mathbb{O}$

(2) if $\mathbb{X} + \mathbb{Y} = \mathbb{O} \rightarrow \mathbb{Y} = -\mathbb{X}$

(3) $(-1)\,\mathbb{X} = -\mathbb{X}$

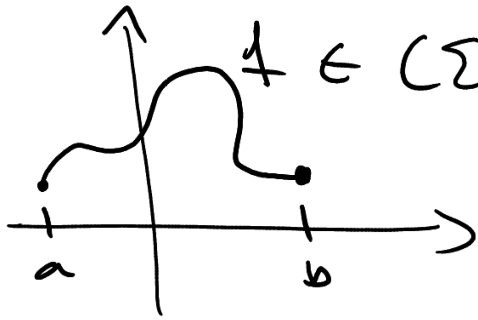
<u>"Vector" Spaces</u>:

any set that has element + element <u>and</u> (scalar)(element) defined <u>and</u> the above 10 properties hold.

$\boxed{\text{ex}}$ $\mathbb{R}^n$ is $\mathbb{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$ with normal $\mathbb{V}_1 + \mathbb{V}_2$, $\alpha \mathbb{V}_1$ defined

$\boxed{ex}$ $C[a,b]$ Vectorspace

element : Continuous function over the interval $[a,b]$



$f \in C[a,b]$

$$\boxed{\begin{array}{l} \underline{\text{define}}: (f+g)(x) = \underline{f(x)+g(x)} \\ \underline{\text{define}}: (\alpha f)(x) = \alpha \underline{f(x)} \end{array}}$$

→ you can show all $\boxed{10}$ properties hold.

P_n polynomial's of n-terms, degree $n-1$

$p(x) \in P_n$ $\quad p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}$

$\underline{\text{define}}$: $(p_1 + p_2)(x) = p_1(x) + p_2(x)$

$\underline{\text{define}}$: $(\alpha p_1)(x) = \alpha p_1(x)$

$R^{m\times n}$ $\quad$ Matrix space:

element is $A = [a_{ij}]$ $m\times n$ matrix

$\underline{\text{define}}$: $A + B$ as normal matrix add

$\underline{\text{define}}$: αA as normal scalar mult.

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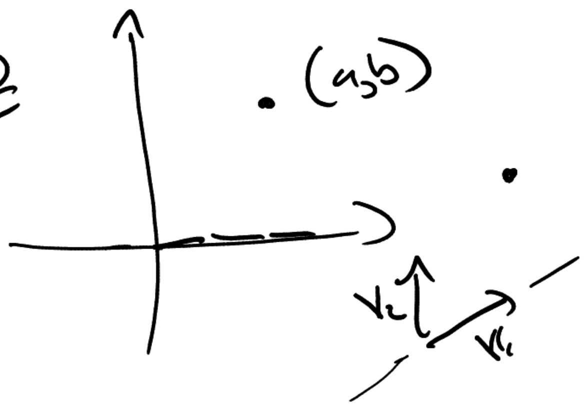
again we can show all 10 properties.

why?

b/c $\underline{e}$ in V given known e_i's

ex $\alpha_1 \underline{e_1} + \alpha_2 \underline{e_2} + \dots + \alpha_n \underline{e_n} = \underline{e}$

2D



$\begin{bmatrix}1\\0\end{bmatrix}$ vs $\begin{bmatrix}0\\1\end{bmatrix}$

$a\begin{bmatrix}1\\0\end{bmatrix} + b\begin{bmatrix}0\\1\end{bmatrix} =$ point (a,b)
