Math 511

Q's / Exam

Fix for 50% missed points

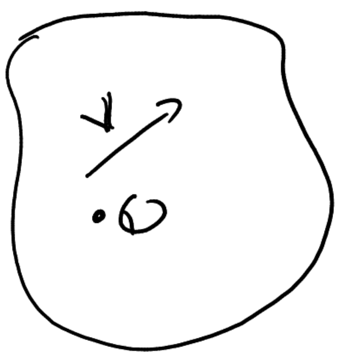
[ex] Name

if you see → -51 $\boxed{69}$

1) -7 ✓ You fix it with no mistake. $+7$

* I'll add these all up to get $\left(\frac{\text{sum}}{2}\right)$ added.

[Ch 3]

S 

Vector Space

elements with $v_1 + v_2$, αv_1

[3.2] Subspace

is a vector space, but it is a subset of another vector space

[ex] $\mathbb{R}^2 \subset \mathbb{R}^3$

2D space is a subspace of $\mathbb{R}^3$

S is a subspace of vector space V ($v_1 + v_2$, αv_1 are defined)

if

[0] S is a non-empty subset of V (at least $0 \in S$)

[1] $v_1 + v_2 \in S$ if $v_1, v_2 \in S$

[2] $\alpha v_1 \in S$ if $v_1 \in S$

So to show S is a subspace you just check

3 things (#1) $0 \overset{?}{\in} S$

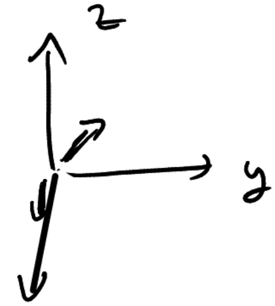
(#2) is $\mathbf{x} + \mathbf{y} \in S$ for $\mathbf{x}, \mathbf{y} \in S$

(#3) is $\alpha \mathbf{x} \in S$

ex: $\mathbb{R}^3$ is my vector space

is $S = \left\{ \begin{bmatrix} a \\ b \\ b \end{bmatrix} \;\middle|\; a, b \text{ are reals} \right\}$

$$\begin{bmatrix} a \\ b \\ b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$



is S a subspace of $\mathbb{R}^3$?

(1) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \overset{?}{\in} S$ $\quad 0 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 0 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\boxed{\text{Yes}}$

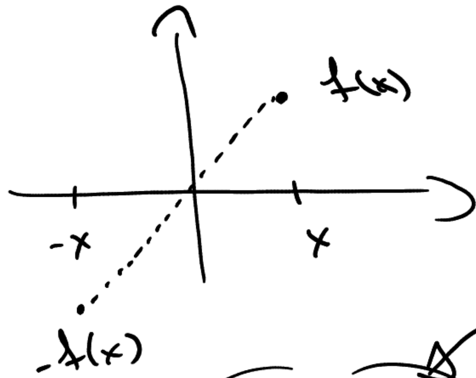
(2) $\begin{bmatrix} a_1 \\ b_1 \\ b_1 \end{bmatrix} + \begin{bmatrix} a_2 \\ b_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 \\ b_1 + b_2 \\ b_1 + b_2 \end{bmatrix}$ same? $\in S$ $\boxed{\text{Yes}}$

(3) $\alpha \mathbf{v}_1 = \begin{bmatrix} \alpha a_1 \\ \alpha b_1 \\ \alpha b_1 \end{bmatrix}$ same? $\boxed{\text{Yes}}$ so $\in S$

$\boxed{S \text{ is a subspace of } V}$

ex $\boxed{C[-1,1]}$ consider $S = \{ f \mid \underline{f \text{ is odd}} \}$

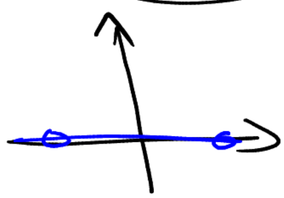
f is odd if $f(-x) = -f(x)$



sym. about origin.

$\boxed{0}$ of $C[-1,1]$

① is $f(x) = 0$ still in S?



$f(-x) = 0 \qquad f(x) = 0$

so $f(-x) = f(x)$

② f and g are $\boxed{\text{odd}}$ (sym. about origin)

is $f+g$ also odd (sym. about origin)?

$(f+g)(-x) = f(-x) + g(-x) = -f(x) + -g(x)$

$= -(f(x) + g(x)) = -(f+g)(x)$

it is odd

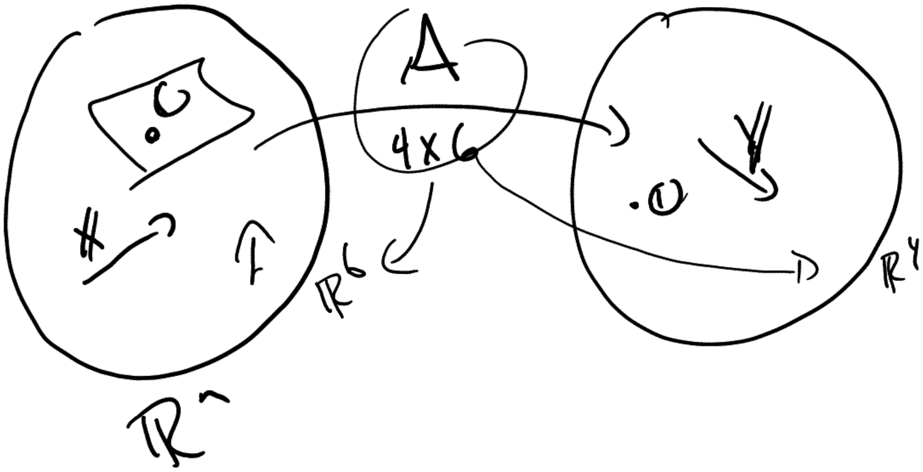
③ $(\alpha f)(-x) = \alpha f(-x) = (\alpha)(-f(x))$

$= -|\alpha f(x)| = -(\alpha f)(x)$

it is odd

ex (important Example)

<u>Consider:</u> $\underset{m\times n}{A}\ \underset{n\times 1}{\mathbb{x}} = \underset{m\times 1}{\mathbb{y}}$ A is $m\times n$



where $A\mathbb{x} = \mathbb{y}$

what are the $\mathbb{x}$'s in $\mathbb{R}^n$ that A maps to the $\mathbb{0}$ in $\mathbb{R}^m$? Collect all $\mathbb{x}$'s in $\mathbb{R}^n$ that

$$A\mathbb{x} = \mathbb{0}$$

$$\underline{N(A)} = \{\mathbb{x} \in \mathbb{R}^n \mid A\mathbb{x} = 0\}$$

"Null Space of A"

Q: is $N(A)$ a subspace of $\mathbb{R}^n$?

① is $\mathbb{0}$ in $N(A)$?

$$A\mathbb{0} = \mathbb{0} \quad \boxed{\text{true}}$$

② $\mathbb{v}_1, \mathbb{v}_2$ in $N(A)$

$$A(\mathbb{v}_1 + \mathbb{v}_2) = A\mathbb{v}_1 + A\mathbb{v}_2 = \mathbb{0} + \mathbb{0} = \mathbb{0}$$

true

③ $\mathbb{v}_1$ in $N(A)$

$$A(\alpha\mathbb{v}_1) = \alpha A\mathbb{v}_1 = \alpha\mathbb{0} = \mathbb{0} \quad \boxed{\text{true}}$$

Why is $N(A)$ important?

Thm $\left(A \text{ is non-singular}\right) \equiv \left(A\mathbf{x} = \mathbf{0} \text{ has } \underline{\text{only}} \text{ trivial soln}\right)$

$N(A)$ is $\underline{\underline{\text{all}}}$ $\mathbf{x}$'s such that $A\mathbf{x} = \mathbf{0}$

$\underline{\text{and}}$ it is a subspace of $\mathbb{R}^n$

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ex $N\left(\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}\right)$

all x's such that $A\mathbf{x} = \mathbf{0}$

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ \boxed{1} & 2 & 3 & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \end{array} \qquad \underbrace{x_2, x_3}_{\underline{\underline{\text{free}}}}$$

free

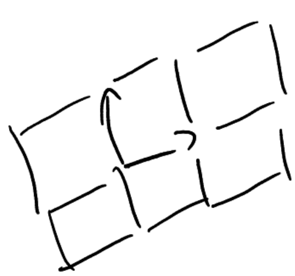
$x_2 = a$

$x_3 = b$

$x_4 = 0$

$x_1 = -2a - 3b$

$$\begin{bmatrix} -2a - 3b \\ 1a + 0b \\ 0a + 1b \\ 0a + 0b \end{bmatrix} = \underbrace{a\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + b\begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \end{bmatrix}}$$



Def: $\mathbb{V}_1, \mathbb{V}_2, \ldots, \mathbb{V}_k \in V$

a <u>linear combination</u> $\alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2 + \cdots + \alpha_k \mathbb{V}_k$

set of <u>all</u> such linear combinations

is called the Span of $\mathbb{V}_1, \mathbb{V}_2, \ldots, \mathbb{V}_k$

<u>Notation:</u> $\text{Span}(\{\mathbb{V}_i\})$

Th^m $\mathbb{V}_i \in V$ then $\text{span}(\{\mathbb{V}_i\})$ is a subspace of V.

<u>Terms:</u> $\text{Span}(\{\mathbb{V}_i\})$ is a set (subspace of V)

so we can say $\mathbb{V}_1, \mathbb{V}_2, \ldots \mathbb{V}_k$ <u>Span</u> this set.

<u>and</u> the set is <u>spanned by</u> $\mathbb{V}_1, \mathbb{V}_2, \ldots, \mathbb{V}_k$

Def $\mathbb{V}_1, \mathbb{V}_2, \ldots, \mathbb{V}_k$ is a spanning set of V, a vector space, iff every vector of V can be written as a linear combo of <u>$\mathbb{V}_i$</u>

(<u>Show:</u> for any arbitrary $\mathbb{V} = \alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2 + \cdots + \alpha_k \mathbb{V}_k$)

$\boxed{3.3}$

$\boxed{ex}$ $\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}$ spans $\mathbb{R}^3$

any V in $\mathbb{R}^3$ $\begin{bmatrix}x\\y\\z\end{bmatrix} = \alpha\begin{bmatrix}1\\0\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\\1\end{bmatrix} + \theta\begin{bmatrix}1\\0\\1\end{bmatrix}$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}\alpha+\theta\\ \beta\\ \beta+\theta\end{bmatrix}$$

$x = \alpha + \theta$

$y = \beta$

$z = \beta + \theta$

$\alpha = x + y - z$

$\beta = y$

$\theta = z - y$

$\begin{bmatrix}1\\1\\1\end{bmatrix} \in \mathbb{R}^3$