Math 511

Q's

3.6 #6

know

① $A\mathbf{x} = \mathbf{b}$ (Solve)

$\mathbf{x} \to x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + \cdots + x_n \mathbf{a}_n$

② $\mathbf{b}$ is col. space of A

any linear combo of the col's of A.

③ $A = [\mathbf{a}_1\ \mathbf{a}_2 \cdots \mathbf{a}_n]$, $\mathbf{a}_i$ are dependent.

Q? how many soln's?

① with ② says we have a soln for $A\mathbf{x} = \mathbf{b}$

$$\boxed{x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + \cdots + x_n \mathbf{a}_n} = \boxed{\mathbf{b}}$$

Note

Th^m $A\mathbf{x} = \mathbf{b}$ has a soln iff $\mathbf{b}$ is in col. space of A.

→ So $A\mathbf{x} = \mathbf{b}$ has 1 or ∞ soln's.

no free vars

have free vars

So ③ we have dependency. → $\boxed{\text{dep. eqn's}}$ → free vrs

→ $\boxed{\infty \text{ solns}}$

3.6 #10

know

① A is $m \times n$ — rows — cols = vars

② $\text{rank}(A) = n$ $\quad$ $\text{rank}(A) = \#$ of lead vars

③ $\boxed{A\mathbf{c} = A\mathbf{d}}$ for some $\mathbf{c}, \mathbf{d}$

Q1 how are $\mathbf{c}, \mathbf{d}$ related (equal, not equal)?

Q2 what about if $\text{rank}(A) < n$?

$\boxed{Q1}$ b/c rank(A) = n → no free vars.

$\mathbb{b} = A\vec{c} = A\vec{d}$ means we have a soln.

→ we can have only $\boxed{1 \text{ soln.}}$

so $\boxed{c = d}$

$\boxed{Q2}$ rank(A) < n → we $\boxed{\text{have free vars.}}$

→ ∞ solns. # of free = nullity = dim(N(A))

Note: $\mathbb{v} \in N(A)$ then $A\mathbb{v} = 0$

if $A\mathbb{x} = \mathbb{b}$

$$A(\mathbb{x}+\mathbb{v}) = A\mathbb{x} + \underbrace{A\mathbb{v}}_{0} = A\mathbb{x} = \mathbb{b}$$

→ $\mathbb{b} = A\mathbb{c} = A\mathbb{d}$ b/c ∞ solns c does not have to be d.

Exam #2

11 probs

$\boxed{3.1}$ Vector Spaces → (objects, scalar mult, add)
(1 prob)

Axioms (I will give these)

① (parts) Are they vector spaces?

$\boxed{3.2}$ Subspaces (1 prob)

Possible
1. $N(A)$
2. subspace of $\mathbb{R}^n$
3. subspace of P_n

$\boxed{1 \text{ (parts)}}$ Are they subspaces?

$\boxed{\text{ex}}$ show $\boxed{N(A)}$ is a subspace.

$$N(A) = \{ v \mid Av = 0 \}$$

1. (does $N(A)$ have 0 in it?)

$$A0 \overset{?}{=} 0 \quad \boxed{\text{yes}}$$

2. (if $v \in N(A)$ is $\alpha v \in N(A)$?)

$$A(\alpha v) = \alpha Av = \alpha 0 = 0 \quad \text{yep!}$$

3. (if $v_1, v_2 \in N(A)$ is $v_1 + v_2 \in N(A)$?)

$$A(v_1 + v_2) = Av_1 + Av_2 = 0 + 0 = 0 \quad \text{yep!}$$

$\boxed{3.3}$ Linear Ind. (2 prob)

1, 2 > are they lin. ind.?

Know:
$\mathbb{R}^n$, P_n,
$C[a,b]$

$\boxed{3.4}$ Basis / Dimensions (1 prob)

1. given vectors $\rightarrow$ find a basis by pare down and/or extending.

$\boxed{\text{ex}}$ vectors $v_1, v_2, \ldots v_6$ for $\mathbb{R}^4$

(basis means we need $\underline{\underline{4}}$ ind. vectors)

$$V = [v_1 \; v_2 \; \ldots \; v_6] \xrightarrow[\text{ops}]{\text{row}} \begin{bmatrix} \boxed{1} & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & \boxed{1} & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} \end{bmatrix}$$ reduced rowech.

$v_2 = 2v_1$

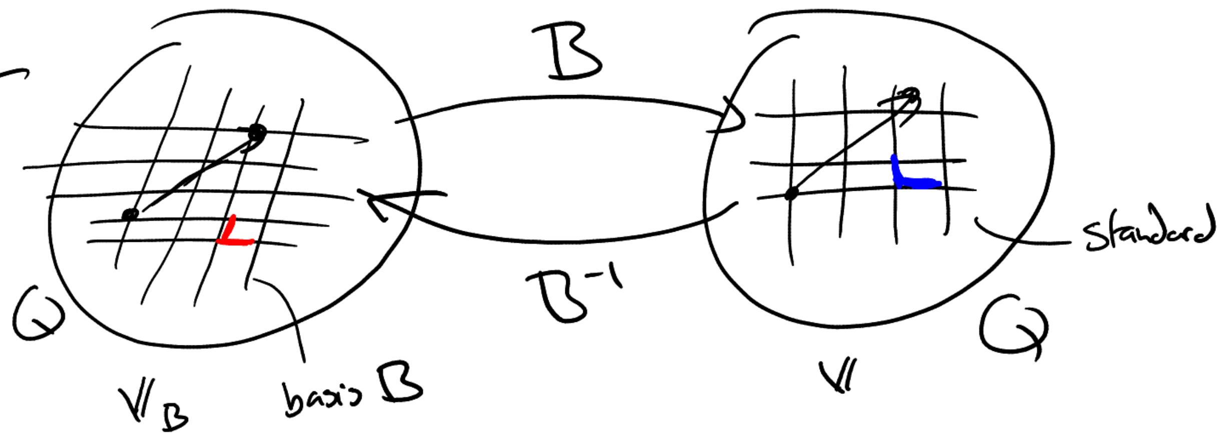
$v_4 = v_1 - v_3$ $\rightarrow$ Keep: $\boxed{v_1, v_3, v_5, v_6}$

$\boxed{3.5}$ Change of Basis (1 prob)

① Find transition matrix for ... ($\mathbb{R}^2$ or P_2 or $C[a,b]$)

$$v = B\,\underline{v_B}$$

Visual idea:



$\boxed{3.6}$ row space / col space (1 prob)

① give you $A \xrightarrow[\text{ops}]{\text{row}} U$ in reduced row ech.

to do: rank(A), nullity, $\boxed{\underset{\uparrow}{N(A)}\text{ span}(\ldots)}$, col space span(..), row space span(--)

to do: dep. eqn's ← use them.

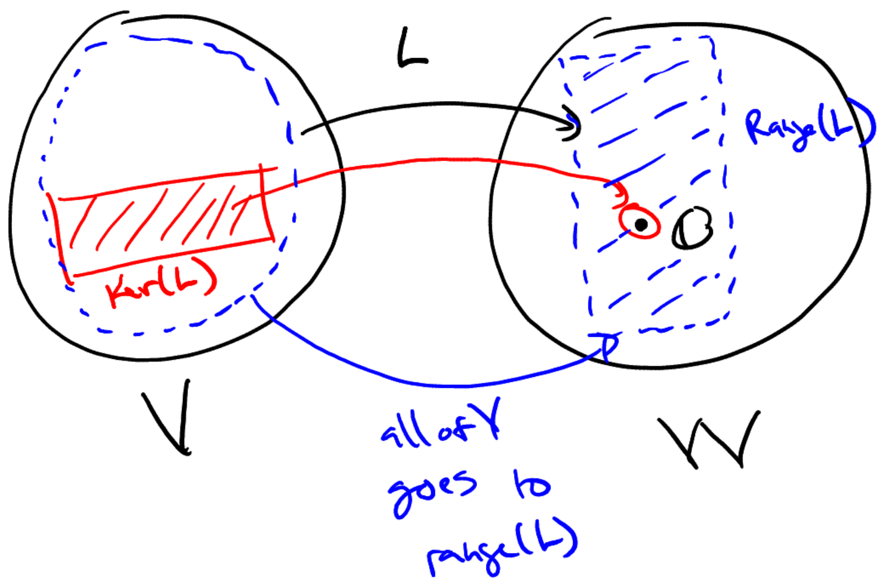
4.1 Linear Trans (2 prob)

① Are they Linear transforms?

Check: $L(a\mathbf{v}_1 + b\mathbf{v}_2) = aL(\mathbf{v}_1) + bL(\mathbf{v}_2)$

② Ker(L)?

range(L)?



4.2 Matrix Rep. of L (2 prob)

① A standard for L

② Matrix from basis V to bases W.

4.3 Similarity (0 prob)